

# Neutrino masses along with fermion mass hierarchy

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## Abstract

Recently a new mechanism has been proposed to cure the problem of fermion mass hierarchy in the Standard Model (SM) model. In this scenario, all SM charged fermions other than top quark arise from higher dimensional operators involving the SM Higgs field. This model also predicted some interesting phenomenology of the Higgs boson. We generalize this model to accommodate neutrino masses ( Dirac & Majorana ) and also obtain the mixing pattern in the leptonic sector. To generate neutrino masses, we add extra three right handed neutrinos ( $N_{iR}$ ) in this model.

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# 1 Introduction

The experimental observations from several neutrino oscillation data indicate that neutrinos have mass of the order of  $\mathcal{O}(10^{-10})$  GeV and they also mix (see [1] and [2] for review). In the Standard Model (SM), all three flavored neutrinos are left handed and massless. Hence, to generate massive neutrinos, one needs to invoke physics beyond the SM. Since neutrinos are electrically neutral, it can be either Dirac or Majorana fermions. To generate the Dirac mass term for the neutrinos, one has to add right handed neutrinos in the particle contents of the SM. On the other hand for Majorana neutrinos, one has to break the lepton number which is an accidental symmetry of the SM.

The seesaw mechanism [3] has been identified as the most natural scenario to generate small masses for neutrinos. In this scenario, one adds a heavy particle of mass  $M$  in the SM, which after being integrated out, leads to the gauge invariant  $D = 5$  operator with only SM fields,  $\mathcal{L}_{eff} = y \frac{LLHH}{M}$ , with  $M \gg M_W$  assumed. There are mainly three types of the seesaw mechanisms have been realized depending upon the type of the exchanged heavy particles:

- Type -I : Heavy right handed neutrinos are exchanged [3, 4]
- Type -II : Heavy  $SU(2)$  triplet scalars are exchanged [5]
- Type -III : Heavy  $SU(2)$  neutral triplet fermions are exchanged [6]

All the above mechanisms have been proposed to generate neutrino masses of the order of  $\mathcal{O}(10^{-10})$  GeV, which immediately suggest that the neutrinos are much lighter than their charged  $SU(2)$  partners. Although, we still do not know the exact masses of the neutrinos, but their mass differences point out of some hierarchies among different generations, which are very different from that of the charged leptons. In addition, the observed neutrino mixing angles indicate strong flavor mixing in the leptonic sector compared to the quark sector. As a result of this, the neutrino mass models are expected to explain not only the smallness of the masses but also the flavor structure of the lepton sector.

It has been shown in Ref. [7] that by using higher dimensional operators involving the relevant SM fermion fields and successive powers of the Higgs doublet field one could obtain a good fit to quark and charged lepton masses and mixing angles. These Yukawa interactions can be expressed in powers of  $H^\dagger H/M^2$ , where  $H$  is the SM Higgs doublet field and  $M$  is a mass scale  $\sim \mathcal{O}(1 - 2)$  TeV at which SM would be embedded in an ultra violet (UV) theory. The dimensionless coefficients of these non-renormalizable operators which has inverse mass-dimensions can all be order one, which in turn leads to a small Yukawa couplings of the SM in a natural way. This model also predicts some interesting Higgs phenomenology, for example, the

enhanced  $b\bar{b}H^0$ , and  $\mu\bar{\mu}H^0$  couplings, flavor changing Higgs boson decay  $H \rightarrow \bar{t}c$  [7]. The strength of the flavor-changing  $\bar{t}cH^0$  vertex is similar in magnitude of the flavor-conserving  $b\bar{b}H^0$  vertex. However, this feature is not visible in the leptonic sector, as the flavor-changing  $\tau\mu H^0$  vertex turns out to be two orders of magnitude suppressed compared to the flavor-conserving  $\tau\tau H^0$  vertex. All these lead to a very interesting phenomenology which can be tested at the ongoing Large Hadron Collider (LHC) experiment.

In Ref. [7], authors have not addressed the issues of neutrino masses. In this paper, we try to obtain the right order of neutrino masses and the corresponding Pontecorvo-Maki-Nakagawa-Sakata (PMNS) [8,9] matrix by introducing three additional right-handed neutrino fields ( $N_{iR}$ ) in the above mentioned model. With these right-handed neutrinos, one can generate both Dirac and Majorana type neutrinos. We then compute the PMNS matrix in both these two cases.

Here, we would like to mention that generation of neutrino masses using effective operators higher than  $d > 5$  have been also discussed in Ref. [10–15]. Most of these models have discussed only neutrino masses, but did not attempt charged fermion masses simultaneously. Moreover, the mixing pattern in lepton sector has also not been addressed in these models. Some of these models have effective  $d = 6, 7$  operators with a TeV scale cutoff to explain the neutrino masses, but at the expense of some more suppression through other couplings. In contrast to the above mentioned models, we are trying to explain neutrino masses and mixing pattern in a model which already has a natural mechanism to explain the charged fermion masses. The effective operators we consider have mass dimensions of 20 or 12, depending on Dirac or Majorana neutrinos, and hence even with a TeV scale cutoff of the model we do not have suppressions in dimensionless couplings. Since the neutrino mass generation in this model correlates with that of charged fermion masses, the parameters of this model are highly constrained, leading to unique predictions for the phenomenology of this model.

The rest of the paper is organized as follows. In Sec.II, we briefly discuss the model of Ref. [7]. In Sec.III we discuss the mechanism of Dirac and Majorana neutrino mass generation. In Sec.IV we discuss the UV completion of the model. In Sec.V we outline some interesting phenomenology of this scenario. Finally, in Sec.VI we summarize our results.

## 2 The Model

In the SM, the top quark mass, whose mass is around the electroweak scale, can be explained naturally by the corresponding Yukawa term. Whereas, the masses for other fermions need suppressions in the respective Yukawa couplings. So in the SM, the hierarchy in mass pattern of fermions reflects into hierarchy in the corresponding

Yukawa couplings. To address this fermion mass hierarchy problem a model has been proposed in [7], where the suppression in Yukawa couplings, other than the top quark, is explained through higher dimensional terms. These higher dimensional terms are expressed in powers of  $\frac{H^\dagger H}{M^2}$ , where  $H$  is the Higgs doublet of the SM and  $M$  is a mass scale at which SM would be embedded in a UV theory. The effective terms in the SM to explain the fermion mass hierarchy are written as [7]

$$\begin{aligned}
\mathcal{L}^{\text{Yuk}} = & h_{33}^u \bar{q}_{3L} u_{3R} \tilde{H} + \left( \frac{H^\dagger H}{M^2} \right) (h_{33}^d \bar{q}_{3L} d_{3R} H + h_{22}^u \bar{q}_{2L} u_{2R} \tilde{H} + h_{23}^u \bar{q}_{2L} u_{3R} \tilde{H} + h_{32}^u \bar{q}_{3L} u_{2R} \tilde{H}) \\
& + \left( \frac{H^\dagger H}{M^2} \right)^2 (h_{22}^d \bar{q}_{2L} d_{2R} H + h_{23}^d \bar{q}_{2L} d_{3R} H + h_{32}^d \bar{q}_{3L} d_{2R} H + h_{12}^u \bar{q}_{1L} u_{2R} \tilde{H} + h_{21}^u \bar{q}_{2L} u_{1R} \tilde{H} \\
& + h_{13}^u \bar{q}_{1L} u_{3R} \tilde{H} + h_{31}^u \bar{q}_{3L} u_{1R} \tilde{H}) + \left( \frac{H^\dagger H}{M^2} \right)^3 (h_{11}^u \bar{q}_{1L} u_{1R} \tilde{H} + h_{11}^d \bar{q}_{1L} d_{1R} H \\
& + h_{12}^d \bar{q}_{1L} d_{2R} H + h_{21}^d \bar{q}_{2L} d_{1R} H + h_{13}^d \bar{q}_{1L} d_{3R} H + h_{31}^d \bar{q}_{3L} d_{1R} H) + \text{h.c.}
\end{aligned} \tag{1}$$

Here,  $h^u$ s and  $h^d$ s are  $\mathcal{O}(1)$  couplings. Also,  $q$ s are left-handed quark doublets,  $u, d$  are singlet right-handed up- and down-type quark fields, respectively.  $\tilde{H}$  is the conjugate of  $H$ . The above higher order terms can be explained from the UV completion of the SM, which will be described later.

Terms in Eq. (1) are higher dimensional and generate effective Yukawa couplings once the Higgs doublet acquires vacuum expectation value (vev). Although the terms in Eq. (1) give masses to quarks, mass generation mechanism for charged leptons is same as that for the down-type quarks. In the above equation by replacing  $q_{iL} \rightarrow L_{iL}$ ,  $u_{iR} \rightarrow E_{iR}$  and  $h_{ij}^d \rightarrow h_{ij}^l$ , where  $L$ s and  $E$ s are left-handed doublet and right-handed singlet leptons, respectively, and  $h^l$ s are  $\mathcal{O}(1)$  couplings, one would obtain mass terms for charged leptons. After the electroweak symmetry breaking the masses of quarks and charged leptons will have a form [7]

$$\begin{aligned}
(m_t, m_c, m_u) & \approx (|h_{33}^u|, |h_{22}^u| \epsilon^2, |h_{11}^u - h_{12}^u h_{21}^u / h_{22}^u| \epsilon^6) v, \\
(m_b, m_s, m_d) & \approx (|h_{33}^d| \epsilon^2, |h_{22}^d| \epsilon^4, |h_{11}^d| \epsilon^6) v, \\
(m_\tau, m_\mu, m_e) & \approx (|h_{33}^l| \epsilon^2, |h_{22}^l| \epsilon^4, |h_{11}^l| \epsilon^6) v,
\end{aligned} \tag{2}$$

where,  $\epsilon = \frac{v}{M}$  and  $v = 174$  GeV is the vev of the Higgs doublet. Along with the mass terms, we can also get Cabbibo-Kobayashi-Maskawa (CKM) matrix in the quark sector. It has been shown in [7] that a good fit to the CKM matrix and to the masses of quarks and charged leptons can be obtained for  $\epsilon = 1/6.5$ , and the various  $\mathcal{O}(1)$

couplings are found out to be<sup>3</sup>

$$\begin{aligned}
(|h_{33}^u|, |h_{22}^u|, |h_{11}^u - h_{12}^u h_{21}^u / h_{22}^u|) &= (0.96, 0.14, 0.95), \\
(|h_{33}^d|, |h_{22}^d|, |h_{11}^d|) &= (0.68, 0.77, 1.65), \\
(|h_{33}^l|, |h_{22}^l|, |h_{11}^l|) &= (0.42, 1.06, 0.21).
\end{aligned} \tag{3}$$

The value  $\epsilon = 1/6.5$  implies that  $M \approx 1.1$  TeV, which is the scale at which a UV completion of the SM takes place.

The UV completion of this model is necessary in order to explain the higher order terms of Eq. (1). Consider a flavor symmetry  $G_F$  above the scale  $M$ , under which the third generation up-quarks and Higgs boson are singlets and all other fermions transform non-trivially. This charge assignment forbids the dimension-4 Yukawa terms for all fermions, except for the top quark. Now, some vector-like heavy fermions and complex scalar flavon fields  $F$  with masses  $\sim M$  can be proposed, which transform under the flavor group  $G_F$  but are singlets under the SM gauge group. The role of these heavy vector-like fermions and flavons  $F$  is such that they form Yukawa-like terms with the SM fermions at a high scale. The flavon fields  $F$  can acquire vev around  $M$  and spontaneously break the flavor symmetry  $G_F$ . Upon integrating the vector-like fermions, we can generate higher dimensional terms of Eq. (1) [7], where the dimensionless couplings  $h^u$ s and  $h^d$ s can be viewed as functions of  $\frac{\langle F \rangle}{M}$ .

It is to be noted that the model in [7] can be generalized by including an additional scalar singlet field  $S$  [16, 17]. It has been shown that instead of expanding in  $\frac{H^\dagger H}{M^2}$ , the higher order terms of Eq. (1) can arise in terms of  $\frac{S^\dagger S}{M^2}$  [16]. The model of this kind is consistent and the UV completion of it has been worked in detail [16]. Likewise, we can also consider higher order terms of Eq. (1) arising in expansion of both  $\frac{H^\dagger H}{M^2}$  and  $\frac{S^\dagger S}{M^2}$  [17]. However, in this work, we stick to the minimal version of all these models [7], i.e. we do not assume extension to scalar Higgs sector of the SM.

### 3 Neutrino masses in this model

In the above described model, neutrino masses have not been addressed, and as a result we cannot also obtain the PMNS matrix in the lepton sector. Here, we address both these issues by proposing three right-handed neutrino fields ( $N_{iR}$ ) into the model. However, right-handed neutrinos can couple to left-handed neutrinos in such a way that either Dirac or Majorana neutrinos can form. We study both these cases in the following two subsections.

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<sup>3</sup>SM fermion masses are given in [7], which include renormalization effects.

### 3.1 Dirac neutrinos

From the neutrino oscillation data it is known that the atmospheric neutrino mass scale ( $\approx 0.05$  eV) is just a factor larger than the solar neutrino mass scale ( $\approx 0.009$  eV). This is to be compared to the hierarchy of  $\sim 10^4$  between electron and tau masses. Indeed, to accommodate hierarchy between different family generations of charged fermions, different powers of  $\frac{H^\dagger H}{M^2}$  are assigned in Eq. (1), which may not be necessary in the case of neutrinos because there are no such large hierarchies within their masses. Hence, from the above mentioned point and also from the naive order of estimations on the neutrino mass scale, we propose the following higher dimensional terms

$$\mathcal{L}_D^\nu = \left( \frac{H^\dagger H}{M^2} \right)^8 h_{ij}^\nu \bar{L}_{iL} N_{jR} \tilde{H}. \quad (4)$$

The mass dimension of the above operators are 20, which is large compared to some dimension-10 operators in the quark sector of Eq. (1). The largeness in the dimension for neutrino operators would give us very small neutrino masses compared to the charged fermion masses. The above higher order terms can be motivated by studying the UV completion of this model, where we appropriately choose the heavy vector-like fermions under the flavor group  $G_F$  and upon integrating them out we generate the above terms in the low energy regime. The UV completion of this model will be described in the next section. After the electroweak symmetry breaking, the above term gives Dirac masses for neutrinos, which has a form

$$[M_D^\nu]_{ij} = \epsilon^{16} v h_{ij}^\nu. \quad (5)$$

Now, our aim is to fit the atmospheric and solar neutrino mass-squared differences and also the PMNS matrix with  $\mathcal{O}(1)$   $h^\nu$  couplings.  $\mathcal{O}(1)$   $h^\nu$  couplings mean that the values should be close to 1, but there is no well defined range for these values. For example, in [12]  $\mathcal{O}(1)$  couplings are meant to be in the range 1/5 to 5. However, in this work we try for  $h^\nu$ s to be between 0.1 and 2.0 because Yukawa couplings for charged fermions are found to be within this range, see Eq. (3). When we present our numerical results we will see that the  $h^\nu$ s may become slightly larger than 2.0 and we comment out over there. As for the PMNS matrix, it has some specific structure and as result we would expect the couplings  $h^\nu$  need to have some structure as well. The PMNS matrix has been defined as  $U_{\text{PMNS}} = (V_L^l)^\dagger V_L^\nu$ , where  $V_L^l$  and  $V_L^\nu$  are unitary matrices which diagonalize the charged lepton and neutrino mass matrices as follows:

$$\begin{aligned} (V_L^l)^\dagger M^l (M^l)^\dagger V_L^l &= \text{diag}(m_e^2, m_\mu^2, m_\tau^2), \\ (V_L^\nu)^\dagger M_D^\nu (M_D^\nu)^\dagger V_L^\nu &= \text{diag}(m_1^2, m_2^2, m_3^2). \end{aligned} \quad (6)$$

Here,  $m_{1,2,3}$  are the three neutrino mass eigenvalues and  $M^l$  is the mass matrix in the

charged lepton flavor basis, whose form is

$$M^l = \begin{pmatrix} h_{11}^l \epsilon^6 & h_{12}^l \epsilon^6 & h_{13}^l \epsilon^6 \\ h_{21}^l \epsilon^6 & h_{22}^l \epsilon^4 & h_{23}^l \epsilon^4 \\ h_{31}^l \epsilon^6 & h_{32}^l \epsilon^4 & h_{33}^l \epsilon^2 \end{pmatrix} v. \quad (7)$$

We have found that the leading form of  $V_L^l$  as

$$V_L^l = \begin{pmatrix} 1 & \frac{h_{12}^l}{h_{22}^l} \epsilon^2 & \frac{h_{13}^l}{h_{33}^l} \epsilon^4 \\ -\frac{h_{12}^l}{h_{22}^l} \epsilon^2 & 1 & \frac{h_{23}^l}{h_{33}^l} \epsilon^2 \\ -\frac{h_{13}^l h_{22}^l - h_{23}^l h_{12}^l}{h_{22}^l h_{33}^l} \epsilon^4 & -\frac{h_{23}^l}{h_{33}^l} \epsilon^2 & 1 \end{pmatrix}. \quad (8)$$

The above equation indicates that the form of  $V_L^l$  is close to unit matrix with the off-diagonal elements are suppressed by at least  $\epsilon^2$ . This observation indicates that the unitary matrix  $V_L^\nu$  should have nearly the PMNS structure.

The PMNS matrix is determined by the three mixing angles and one CP violating phase. In this work, for parameterization of PMNS matrix we have followed the convention in [18]. Before the data of T2K experiment, a global fit to various neutrino oscillation data [19] gave results that the  $\theta_{13}$  was allowed to be zero at  $2\sigma$  level and the exact tribimaximal mixing pattern [20] in the lepton sector was still a possibility. Recently, in the T2K experiment [21] the appearance of six events of electron-neutrinos in the detector has ruled out  $\theta_{13} \neq 0$  at 90 % C.L. However, the analysis of T2K is done by putting  $\theta_{12} \approx 34^\circ$  and  $\theta_{23} = 45^\circ$ , which suggests that the values of  $\theta_{12}$  and  $\theta_{23}$  are in agreement with the corresponding tribimaximal values. To be consistent with the T2K experimental result, we take the CP violating phase to be zero and the leptonic mixing angles to be:  $\sin \theta_{12} = \frac{1}{\sqrt{3}}$ ,  $\sin \theta_{23} = \frac{1}{\sqrt{2}}$ , and  $\sin \theta_{13} = 0.157$ . The  $\sin \theta_{13}$  value gives  $\theta_{13} \approx 9^\circ$ , which is consistent with the lower and upper bounds by the T2K [21] and CHOOZ experiments [22], respectively. We consider this value for  $\theta_{13}$  only to demonstrate that PMNS structure can be obtained with  $\mathcal{O}(1)$  Yukawa couplings, but otherwise it can be varied within the experimental limits.

We take the unitary matrix  $V_L^\nu$  as

$$V_L^\nu = \begin{pmatrix} \sqrt{\frac{2}{3}} c_{13} & \frac{1}{\sqrt{3}} c_{13} & s_{13} \\ -\frac{1}{\sqrt{6}} - \frac{1}{\sqrt{3}} s_{13} & \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{6}} s_{13} & \frac{1}{\sqrt{2}} c_{13} \\ \frac{1}{\sqrt{6}} - \frac{1}{\sqrt{3}} s_{13} & -\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{6}} s_{13} & \frac{1}{\sqrt{2}} c_{13} \end{pmatrix}, \quad (9)$$

where  $c_{13} = \cos \theta_{13}$ ,  $s_{13} = \sin \theta_{13}$ . The above form of  $V_L^\nu$  is same as the PMNS matrix with mixing angles in the leptonic sector, as mentioned above. Using the above  $V_L^\nu$ ,

we find

$$(V_L^\nu)^\dagger M_D^\nu (M_D^\nu)^\dagger V_L^\nu = \epsilon^{32} v^2 \begin{pmatrix} \vec{a} \cdot \vec{a} & \vec{a} \cdot \vec{b} & \vec{a} \cdot \vec{c} \\ \vec{a} \cdot \vec{b} & \vec{b} \cdot \vec{b} & \vec{b} \cdot \vec{c} \\ \vec{a} \cdot \vec{c} & \vec{b} \cdot \vec{c} & \vec{c} \cdot \vec{c} \end{pmatrix}, \quad (10)$$

where the 3-dimensional vectors are:  $\vec{a} = (a_1, a_2, a_3)$ ,  $\vec{b} = (b_1, b_2, b_3)$  and  $\vec{c} = (c_1, c_2, c_3)$ , with

$$\begin{aligned} a_j &= \sqrt{\frac{2}{3}} c_{13} h_{1j}^\nu - \left( \frac{1}{\sqrt{6}} + \frac{1}{\sqrt{3}} s_{13} \right) h_{2j}^\nu + \left( \frac{1}{\sqrt{6}} - \frac{1}{\sqrt{3}} s_{13} \right) h_{3j}^\nu, \\ b_j &= \frac{1}{\sqrt{3}} c_{13} h_{1j}^\nu + \left( \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{6}} s_{13} \right) h_{2j}^\nu - \left( \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{6}} s_{13} \right) h_{3j}^\nu, \\ c_j &= s_{13} h_{1j}^\nu + \frac{1}{\sqrt{2}} c_{13} (h_{2j}^\nu + h_{3j}^\nu). \end{aligned} \quad (11)$$

The necessary conditions to be satisfied in order to fit the neutrino mass-square differences are

$$\begin{aligned} m_1^2 &= f_D^2 \vec{a} \cdot \vec{a}, \quad m_2^2 = f_D^2 \vec{b} \cdot \vec{b}, \quad m_3^2 = f_D^2 \vec{c} \cdot \vec{c}, \\ \vec{a} \cdot \vec{b} &= \vec{b} \cdot \vec{c} = \vec{a} \cdot \vec{c} = 0, \\ m_3^2 - m_1^2 &= \Delta m_{\text{atm}}^2 = (\pm 2.4) \times 10^{-3} \text{ eV}^2, \\ m_2^2 - m_1^2 &= \Delta m_\odot^2 = 7.6 \times 10^{-5} \text{ eV}^2, \end{aligned} \quad (12)$$

where  $f_D^2 = \epsilon^{32} v^2$ .

In Eq. (12) we take the central values of the atmospheric and solar neutrino mass-squared differences as they are considered in T2K experiment [21]. Since the sign of  $\Delta m_{\text{atm}}^2$  is not known in experiments, its value could be either of the values given above. We can take the form the vectors as  $\vec{a} = |\vec{a}|(1, 0, 0)$ ,  $\vec{b} = |\vec{b}|(0, 1, 0)$  and  $\vec{c} = |\vec{c}|(0, 0, 1)$ , to satisfy the orthogonality among these vectors. The magnitude of these vectors ( $|\vec{a}|, |\vec{b}|, |\vec{c}|$ ) can be determined by fitting to the neutrino mass-squared differences. If the sign of  $\Delta m_{\text{atm}}^2$  is positive, we get the normal hierarchical pattern among the neutrino masses. In this case we can take  $m_1 \sim \sqrt{\Delta m_\odot^2}$ ,  $m_2 = \sqrt{\Delta m_\odot^2 + m_1^2}$ , and  $m_3 = \sqrt{\Delta m_{\text{atm}}^2 + m_1^2}$ . Whereas, in the case where the sign of  $\Delta m_{\text{atm}}^2$  is negative, the spectrum of neutrino mass eigenstates is called inverted hierarchy, in which case we can take  $m_3 \sim \sqrt{\Delta m_{\text{atm}}^2}$ ,  $m_1 = \sqrt{\Delta m_{\text{atm}}^2 + m_3^2}$ , and  $m_2 = \sqrt{\Delta m_\odot^2 + m_1^2}$ . For a definite value of  $\epsilon$ , and after finding the values of  $|\vec{a}|$ ,  $|\vec{b}|$  and  $|\vec{c}|$ , by inverting Eq. (11), we compute the full neutrino Yukawa couplings.

In the determination of  $|\vec{a}|$ ,  $|\vec{b}|$  and  $|\vec{c}|$ , the value of  $\epsilon$  should be fixed. We fix  $\epsilon = 1/6.5 \approx 0.15$  as this value has given a good fit to charged fermion masses and



also the CKM matrix. In the normal hierarchy, for  $m_1 = 0$  and  $m_1 = 0.7 \times \sqrt{\Delta m_\odot^2}$ , the Yukawa couplings, respectively, turns out to be

$$h^\nu = \begin{pmatrix} 0 & 0.29 & 0.45 \\ 0 & 0.26 & 1.99 \\ 0 & -0.33 & 1.99 \end{pmatrix}, \quad \begin{pmatrix} 0.29 & 0.35 & 0.45 \\ -0.18 & 0.32 & 2.01 \\ 0.11 & -0.40 & 2.01 \end{pmatrix}. \quad (13)$$

In the above case, if  $m_1$  is less than 0.7 times of  $\sqrt{\Delta m_\odot^2}$ , the element  $h_{31}^\nu$  may become less than 0.1. In the inverted hierarchy, for  $m_3 = 0$  and  $m_3 = 0.5 \times \sqrt{\Delta m_{\text{atm}}^2}$ , the Yukawa couplings, respectively, come out to be

$$h^\nu = \begin{pmatrix} 2.31 & 1.66 & 0 \\ -1.43 & 1.49 & 0 \\ 0.91 & -1.86 & 0 \end{pmatrix}, \quad \begin{pmatrix} 2.58 & 1.84 & 0.22 \\ -1.59 & 1.66 & 0.99 \\ 1.01 & -2.08 & 0.99 \end{pmatrix}. \quad (14)$$

The above neutrino Yukawa couplings are  $\mathcal{O}(1)$ . As stated before, compared to charged fermion Yukawa couplings whose values are within 0.1 to 2.0, some elements of neutrino Yukawa couplings are slightly above 2.0, especially in the inverted hierarchical case. We have found that by increasing  $\epsilon$  value to 0.16, the neutrino Yukawa couplings would be in the range of 0.1 to 2.0 in both the hierarchical cases. By increasing  $\epsilon$  to 0.16, do not change the charge fermion Yukawa couplings very much. In fact, we have also checked that for  $\epsilon = 0.16$  the CKM matrix elements can be fitted to their experimental values. We will comment more on the possible values of  $\epsilon$  in Sec. 5, where we describe upper limits on it arising due to  $D^0 - \bar{D}^0$  mixing. Finally, we compute the PMNS matrix which is given by  $(V_L^l)^\dagger V_L^\nu$ . This matrix depends on the charged lepton Yukawa couplings, for which only the diagonal elements have been computed from their mass relations. If we set all the off-diagonal couplings for charged leptons to be 0.5, the actual PMNS matrix in this model for  $\epsilon = 1/6.5$  is

$$U_{\text{PMNS}} = \begin{pmatrix} 0.81 & 0.56 & 0.15 \\ -0.50 & 0.54 & 0.68 \\ 0.30 & -0.63 & 0.72 \end{pmatrix}. \quad (15)$$

The above matrix elements are within the error limits of the computed values through the mixing angles in neutrino oscillation experiments. By changing the off-diagonal Yukawa couplings of charged leptons to 1.5, we have found that the elements in the above matrix will change only in the second decimals.

### 3.2 Majorana neutrinos

In the previous subsection we have analyzed the case of Dirac neutrinos in the model [7]. In the case of Dirac neutrinos the fields  $N_{iR}$  are just the right-handed components

of neutrinos and lepton number is conserved. However, the phenomenology would change if the lepton number is assumed to be violated, and we can have massive sterile neutrinos in the model. Since the UV cutoff of the model is  $M \sim \text{TeV}$ , we can choose the masses of the sterile neutrinos to be slightly less than of the order of TeV. These TeV sterile neutrinos, with some mixing with active neutrinos, can give significant contribution to neutrinoless double beta decay process and some collider processes as well.

Suppose that the following effective operators exist in the model.

$$\mathcal{L}_M = \left( \frac{H^\dagger H}{M^2} \right)^4 h_{ij}^\nu \bar{L}_{iL} N_{iR} \tilde{H} + \frac{M_i}{2} \overline{N_{iR}^c} N_{iR} + \text{h.c.}, \quad (16)$$

where  $M_i$ ,  $i = 1, 2, 3$ , are the masses of the right-handed sterile neutrinos, and the indices  $i, j$  should be summed over 1,2,3.  $N_{iR}^c$  is the charge conjugate of  $N_{iR}$ . There are dimension-12 operators in the above equation which can be motivated from the UV completion of this model, and it will be presented in the next section. The masses of right-handed neutrinos would be around TeV which is much larger than the Dirac neutrino masses of the first term in the above equation. Hence, after integrating the heavy right-handed neutrinos, the masses of light neutrinos are given by

$$(M_M^\nu)_{ij} = \epsilon^{16} v^2 \sum_{k=1}^3 h_{ik}^\nu \frac{1}{M_k} h_{jk}^\nu. \quad (17)$$

Since the above mass matrix is symmetric, it can be diagonalized by a unitary matrix  $V_L^\nu$  as

$$(V_L^\nu)^\text{T} M_M^\nu V_L^\nu = \text{diag}(m_1, m_2, m_3). \quad (18)$$

Since we have argued previously that the corresponding unitary matrix  $V_L^l$  in the charged lepton sector is close to unit matrix, we choose  $V_L^\nu$  to have the form in Eq. (9). Then the above matrix relation can be satisfied if the following relations are hold:

$$\begin{aligned} m_1 &= f_M \vec{a}' \cdot \vec{a}', \quad m_2 = f_M \vec{b}' \cdot \vec{b}', \quad m_3 = f_M \vec{c}' \cdot \vec{c}', \\ \vec{a}' \cdot \vec{b}' &= \vec{a}' \cdot \vec{c}' = \vec{b}' \cdot \vec{c}' = 0, \end{aligned} \quad (19)$$

where  $f_M = \epsilon^{16} v^2$ . The vectors  $\vec{a}' = (a'_1, a'_2, a'_3)$ ,  $\vec{b}' = (b'_1, b'_2, b'_3)$  and  $\vec{c}' = (c'_1, c'_2, c'_3)$  are such that  $a'_i = \frac{a_i}{\sqrt{M_i}}$ ,  $b'_i = \frac{b_i}{\sqrt{M_i}}$  and  $c'_i = \frac{c_i}{\sqrt{M_i}}$ , where  $a_i, b_i, c_i$  are defined in Eq. (11). To satisfy the orthogonality in these vectors we choose their forms as  $\vec{a}' = |\vec{a}'|(1, 0, 0)$ ,  $\vec{b}' = |\vec{b}'|(0, 1, 0)$ ,  $\vec{c}' = |\vec{c}'|(0, 0, 1)$ . Like in the previous Dirac neutrinos, in this case also to fit the neutrino mass-squared differences we analyze both the normal and inverted hierarchical mass spectrums of neutrinos. The mass eigenvalues of the three

neutrinos in terms of mass-squared differences are mentioned in the previous subsection. After finding the  $|\vec{a}'|, |\vec{b}'|, |\vec{c}'|$  and for definite values of right-handed neutrino masses, we can invert the relations in Eq. (11) to find the Yukawa couplings. The masses of right-handed neutrinos could be either degenerate or non-degenerate, and so we study both these cases below. In our numerical analysis we have fixed  $\epsilon = 1/6.5$ .

### Case I: Degenerate right-handed neutrinos

We choose the degenerate mass scale of right-handed neutrinos to be 1 TeV, i.e.  $M_1 = M_2 = M_3 = 1$  TeV. In the normal hierarchy, for  $m_1 = 0$  and  $m_1 = 0.5 \times \sqrt{\Delta m_\odot^2}$ , the Yukawa couplings came out to be, respectively, as

$$h^\nu = \begin{pmatrix} 0 & 0.98 & 0.64 \\ 0 & 0.88 & 2.83 \\ 0 & -1.10 & 2.83 \end{pmatrix}, \quad \begin{pmatrix} 0.98 & 1.03 & 0.64 \\ -0.60 & 0.93 & 2.84 \\ 0.38 & -1.16 & 2.84 \end{pmatrix}. \quad (20)$$

In the inverted hierarchy, for  $m_3 = 0$  and  $m_3 = 0.5 \times \sqrt{\Delta m_{\text{atm}}^2}$ , the Yukawa couplings came out to be, respectively, as

$$h^\nu = \begin{pmatrix} 3.27 & 2.33 & 0 \\ -2.02 & 2.10 & 0 \\ 1.29 & -2.62 & 0 \end{pmatrix}, \quad \begin{pmatrix} 3.46 & 2.46 & 0.45 \\ -2.14 & 2.21 & 2.00 \\ 1.36 & -2.77 & 2.00 \end{pmatrix}. \quad (21)$$

Notice here that in the inverted hierarchical case, the element  $h_{11}^\nu$  of neutrino Yukawa coupling is even larger than 3.0. This situation can be improved if we raise  $\epsilon$  to 0.16, in which case  $h_{11}^\nu$  would be between 2.0 and 3.0. Further increasing  $\epsilon$  to 0.17 would bring all the neutrino Yukawa couplings in the range of 0.1 to 2.0 in both the hierarchical cases. However,  $\epsilon = 0.17$  may be ruled out by the  $D^0 - \bar{D}^0$  mixing in this model, which will be described in Sec. 5.

### Case II: Non-degenerate right-handed neutrinos

To illustrate how the neutrino Yukawa couplings change from Case I, we take the following values for the three right-handed neutrinos:  $M_1 = 500$  GeV,  $M_2 = 800$  GeV and  $M_3 = 1$  TeV. In the normal hierarchy, for  $m_1 = 0$  and  $m_1 = 0.5 \times \sqrt{\Delta m_\odot^2}$ , the Yukawa couplings came out to be, respectively, as

$$h^\nu = \begin{pmatrix} 0 & 0.87 & 0.64 \\ 0 & 0.78 & 2.83 \\ 0 & -0.98 & 2.83 \end{pmatrix}, \quad \begin{pmatrix} 0.69 & 0.92 & 0.64 \\ -0.43 & 0.83 & 2.84 \\ 0.27 & -1.04 & 2.84 \end{pmatrix}. \quad (22)$$

In the inverted hierarchy, for  $m_3 = 0$  and  $m_3 = 0.5 \times \sqrt{\Delta m_{\text{atm}}^2}$ , the Yukawa couplings

came out to be, respectively, as

$$h^\nu = \begin{pmatrix} 2.31 & 2.08 & 0 \\ -1.43 & 1.88 & 0 \\ 0.91 & -2.34 & 0 \end{pmatrix}, \quad \begin{pmatrix} 2.44 & 2.20 & 0.45 \\ -1.51 & 1.98 & 2.00 \\ 0.96 & -2.47 & 2.00 \end{pmatrix}. \quad (23)$$

Comparing the values of neutrino Yukawa couplings in this case with that of Case I, we can notice that by decreasing the value of a right-handed neutrino mass decreases the elements in the corresponding column of the matrix  $h^\nu$ . For example, by decreasing the value of  $M_1$  from 1 TeV to 500 GeV, we can see that the elements in the first column of  $h^\nu$  have decreased. This fact can be understood from the expressions of  $a'_i, b'_i, c'_i$  and also the form of these vectors that we have considered.

Finally, the PMNS matrix in the Majorana neutrino case is same as that of Eq. (15). The construction of PMNS matrix in this model is such that it depends on  $\epsilon$ , charged lepton Yukawa couplings and mixing angles which we have mentioned before. As a result of this, either in the Case I or Case II of Majorana neutrinos, we do get the same PMNS matrix as long as we do not change the  $\epsilon$  and charge lepton Yukawa couplings. This suggests that in this model the right-handed neutrino masses do not show up in the PMNS matrix, rather their implications are felt in the neutrino Yukawa couplings.

## 4 UV completion of the model

In this section we describe how the higher order terms of Eqs. (4) and (16) can be motivated from the UV completion of the model. Our procedure of UV completion is similar to that described in [7, 16, 17]. However, here we do not fix the charges of any field and moreover we study the higher order terms of any power in  $\frac{H^\dagger H}{M^2}$ . In the case of Dirac and Majorana neutrinos of this model, we need 8 and 4 powers of  $\frac{H^\dagger H}{M^2}$ , respectively, in the relevant higher dimensional operators. In our approach, we first study how a single power of  $\frac{H^\dagger H}{M^2}$  is possible in a higher dimensional operator of  $\frac{H^\dagger H}{M^2} \bar{L}_L N_R \tilde{H}$ . For simplicity, here we have suppressed family indices. Then in the next step, we try to see how two powers of  $\frac{H^\dagger H}{M^2}$  can arise in the  $\left(\frac{H^\dagger H}{M^2}\right)^2 \bar{L}_L N_R \tilde{H}$ . Then afterwards, we can generalize this procedure to obtain any higher dimensional operator containing a finite power of  $\frac{H^\dagger H}{M^2}$ . Since the origin of higher order terms in quark and charged lepton sectors have already been discussed in [7, 16, 17], below we confine only to the neutrino sector. However, we believe our procedure, with a little modification, can be merged with that of [7, 16, 17]. Otherwise, our procedure can be extended even to the quark and charged lepton sectors.

As stated before, we have to propose a flavor symmetry group  $G_F$  to forbid the leading Yukawa couplings in the neutrino sector. The symmetry  $G_F$  is an exact sym-

metry at and above the scale  $M \sim \text{TeV}$ . We take  $G_F$  to be a gauged abelian symmetry, i.e.  $G_F = \text{U}(1)_F$ . In a first step to generate the higher order term  $\frac{H^\dagger H}{M^2} \bar{L}_L N_R \tilde{H}$ , we propose vector-like, color-singlet fermionic fields  $K_1$  and  $G_1$ , which are singlet and doublet, respectively, under the  $\text{SU}(2)_L$  of the SM gauge group. We also propose complex scalar flavon field  $F_1$ , whose vev ( $\langle F_1 \rangle \sim M$ ) spontaneously breaks the flavor symmetry  $\text{U}(1)_F$ , but otherwise is singlet under the standard model gauge group. Now, consider the following renormalizable terms at the high scale.

$$y_1 \bar{L}_L K_{1R} \tilde{H} + y_2 F_1 \bar{K}_{1R} K_{1L} + y_3 \bar{K}_{1L} G_{1R} H^\dagger + y_4 F_1 \bar{G}_{1R} G_{1L} + y_5 \bar{G}_{1L} N_R H, \quad (24)$$

where  $y_s$  are  $\mathcal{O}(1)$  dimensionless couplings.

The above renormalizable terms can be justified by assigning the following charges under  $\text{U}(1)_F$ :  $L_L = K_{1R} = l$ ,  $F_1 = f_1$ ,  $K_{1L} = G_{1R} = l - f_1$ ,  $G_{1L} = N_R = l - 2f_1$ ,  $H = 0$ . Here, the Higgs doublet is uncharged under  $\text{U}(1)_F$  and  $l$  and  $f_1$  are  $\text{U}(1)_F$  charges of lepton doublet and  $F_1$ , respectively. After spontaneous breaking of  $\text{U}(1)_F$  and after integrating out the heavy fermions ( $K_1, G_1$ ), the above terms generate  $\frac{H^\dagger H}{M^2} \bar{L}_L N_R \tilde{H}$  in the low energy regime. Notice here that by assigning different  $\text{U}(1)_F$  charges of the three lepton doublets and by proposing three different copies of  $F_1$  field, i.e.  $F_{1i}$ ,  $i = 1, 2, 3$ , the above procedure can be generalized to give the full  $3 \times 3$  family structure in  $\frac{H^\dagger H}{M^2} \bar{L}_L N_R \tilde{H}$ . In this generalization, we need three copies of  $K_1, G_1$  and moreover all the three right-handed neutrinos will have same  $\text{U}(1)_F$  charge.

Let us note here that the field content proposed in Eq. (24) somewhat resembles to that of [23], where a general study of seesaw Dirac neutrino masses has been studied with arbitrary number of active and sterile neutrinos. Here the weak-singlet fields  $N_R, K_{1L}, K_{1R}$  are sterile and they have mixing masses with the active neutrino ( $\nu_L$ ) of  $L_L$ . The mixing between  $K_{1L}$  and  $N_R$  arises after integrating the heavy weak-doublet  $G$ . As a result, we get  $4 \times 4$  mixing mass matrix among the above said fields. The structure of this matrix is same as that of  $4 \times 4$  texture proposed in [23]. In another context of these type of extended seesaw models, leptogenesis has also been studied [24].

In the next step, to generate the higher order term  $\left(\frac{H^\dagger H}{M^2}\right)^2 \bar{L}_L N_R \tilde{H}$ , in addition to the above field content, we propose  $K_2, G_2$  whose quantum numbers under SM gauge group are same as that of  $K_1, G_1$ , and also a complex scalar flavon field  $F_2$ , which is a singlet under the standard model gauge group. Now, consider the following renormalizable terms at the high scale.

$$y_1 \bar{L}_L K_{1R} \tilde{H} + y_2 F_1 \bar{K}_{1R} K_{1L} + y_3 \bar{K}_{1L} G_{1R} H^\dagger + y_4 F_2 \bar{G}_{1R} G_{1L} + y_5 \bar{G}_{1L} K_{2R} H \\ + y_6 F_2 \bar{K}_{2R} K_{2L} + y_7 \bar{K}_{2L} G_{2R} H^\dagger + y_8 F_1 \bar{G}_{2R} G_{2L} + y_9 \bar{G}_{2L} N_R H. \quad (25)$$

Here,  $y_s$  are  $\mathcal{O}(1)$  dimensionless couplings. The above terms can be justified with the following  $\text{U}(1)_F$  charges:  $L_L = K_{1R} = l$ ,  $F_1 = f_1$ ,  $K_{1L} = G_{1R} = l - f_1$ ,  $F_2 = f_2$ ,  $G_{1L} =$

$K_{2R} = l - f_1 - f_2, K_{2L} = G_{2R} = l - f_1 - 2f_2, G_{2L} = N_R = l - 2f_1 - 2f_2, H = 0$ . After spontaneous symmetry breaking and also after integrating the heavy fields we can generate  $\left(\frac{H^\dagger H}{M^2}\right)^2 \bar{L}_L N_R \tilde{H}$  in the low energy regime. Again, to generate the full  $3 \times 3$  family structure in this higher dimensional operator, we have to propose three copies of  $F_1, K_1, K_2, G_1, G_2$ , like we have explained previously. It can be noticed that the charge assignment and field content is such that, while generating  $\left(\frac{H^\dagger H}{M^2}\right)^2 \bar{L}_L N_R \tilde{H}$ , the other higher dimensional term  $\frac{H^\dagger H}{M^2} \bar{L}_L N_R \tilde{H}$  can not be possible in the low energy regime.

Now, it is easy to understand that the above described procedure can be generalized to obtain some finite power of  $\frac{H^\dagger H}{M^2}$  along with the  $\bar{L}_L N_R \tilde{H}$ . To generate the higher dimensional term of Eq. (4), we need the following eight different heavy fields: complex scalar flavon fields ( $F_i, i = 1, \dots, 8$ ), singlet and doublet of  $SU(2)_L$  of the SM gauge group ( $K_i, G_i, i = 1, \dots, 8$ ). Also, to get the full  $3 \times 3$  family structure in the  $\bar{L}_L N_R \tilde{H}$ , we have to replicate the above heavy fields, with different charges, three times. The number of heavy fields that we have described in the case of Dirac neutrinos will be reduced in the case of Majorana neutrinos. To generate the first term of Eq. (16), we have to propose four different heavy fields of scalar and fermionic type, which we have described above. On top of this, to generate the second term of Eq. (16), we have to propose additional complex scalar field with a charge of  $-2n$ , where  $n$  is the  $U(1)_F$  charge of the field  $N_R$ .

## 5 Phenomenology

The phenomenology of this model in the quark and charged lepton sector is same as that described in [7]. In the neutrino sector we get new phenomenological signals which will be described below.

First, from the UV completion of the higher dimensional operators of Eqs. (4) and (16), we need some heavy weak-singlet and weak-doublet fermionic states. In the case of Dirac neutrinos, the number of these heavy fermionic states is  $8 \times 3$  of both weak-singlet and weak-doublet fields. Whereas in the Majorana case this number is  $4 \times 3$ . The masses of these heavy fermionic fields are in the range of 1–2 TeV. The heavy weak-singlet  $K$ s have zero hypercharge, whereas the weak-doublet fields ( $G$ s) have hypercharge  $+1$ . Hence, the heavy weak-doublet fields can be produced either through  $W$  or  $Z$  boson fusion at the LHC. However, detection of weak-singlets is challenging due to its sterile nature. These weak-singlet fermions can be produced in a collider process through the decay of heavy weak-doublet fermions. One of the weak-singlets ( $K_1$  in the previous section) is bound to decay into an active neutrino and Higgs boson. And similarly, one of the weak-doublets ( $G_1$  or  $G_2$  of the previous section) is bound to decay into a right-handed neutrino and Higgs boson state, if

this is kinematically allowed. As a result of these interactions, after the weak-singlet and weak-doublet fermions being produced in a collider, they can ultimately cascade down to some number of neutrinos and Higgs bosons, depending on the masses of these fermions. In this model there are also heavy complex scalar fields, which are SM gauge singlets and have masses of  $\sim 1$  TeV. The complex scalar flavon fields ( $F_i$ ), apart from their Yukawa couplings to heavy fermions, can have quartic interactions with the Higgs doublet field. As a result of this, each scalar flavon field can decay to a pair of Higgs bosons. These complex scalar fields can be produced through the decay of either weak-singlet or weak-doublet heavy fermion.

Apart from the phenomenology of heavy fermions and scalars in the model, the local nature of the flavor group  $U(1)_F$  give some more phenomenology. Since the flavor group is gauged, there would be a gauge boson  $Z'$  corresponding to the  $U(1)_F$ . The mass of  $Z'$  could be in the TeV scale provided the gauge coupling ( $g_F$ ) of  $U(1)_F$  is  $\mathcal{O}(1)$ . In the UV completion of this model we have assumed that leptons are charged under  $U(1)_F$  and without any inconsistency we can assume that quarks are singlets under  $U(1)_F$ . This particular charge assignment can relax constraints on the gauge coupling  $g_F$  due to Drell-Yan process. However, the charge assignment of leptons can induce mixing between  $Z$  and  $Z'$  at a loop level. Since this mixing should be small [18], we may have to suppress the coupling  $g_F$ . The details of these studies is beyond the limit of this work, but we refer some recent works on the phenomenology of  $Z'$  [25]. It is to be noticed that some of the phenomenology of  $Z'$  studied in [16] can also be applicable in this model. Finally, the heavy fermions are chiral with respect to the flavor group and as a result there could be anomalies due to gauged  $U(1)_F$ . To cancel these anomalies we have to propose some additional fermions at the TeV scale through Green-Schwarz mechanism, which is also suggested in [16].

Next, we focus on the phenomenology arising due to neutrino masses of this model. As said previously, one of the consequences of Majorana neutrinos is that it generates neutrinoless double beta decay process at tree level. This has been looked in various experiments, see Refs. [26], for conducted and future proposed experiments. The amplitude of this process depends on the quantity  $m_{ee} = \sum_{i=1}^3 U_{ei}^2 m_i$  [27], where  $U = U_{\text{PMNS}}$  and  $m_i$  are the mass eigenvalues of light neutrinos. The non-observation of this process has put an upper bound on  $m_{ee}$  to be  $\sim 0.5$  eV [28]. In our particular case of Majorana neutrinos, for  $\epsilon = 1/6.5$ , we have calculated the  $m_{ee}$  of neutrinoless double beta decay process. In the normal hierarchy, for  $m_1 = 0$  and  $m_1 = 0.5 \times \sqrt{\Delta m_{\odot}^2}$ ,  $m_{ee}$  is  $3.9 \times 10^{-3}$  eV and  $7.1 \times 10^{-3}$  eV, respectively. In the inverted hierarchy, for  $m_3 = 0$  and  $m_3 = 0.5 \times \sqrt{\Delta m_{\text{atm}}^2}$ ,  $m_{ee}$  is found out to be 0.048 eV and 0.054 eV, respectively. Note here that we put Majorana phases to be zero in this calculation. An interesting fact is that the values we get for the quantity  $m_{ee}$  is independent of right-handed neutrino masses, since they do not enter in the construction of the PMNS matrix of this model.

Nonzero masses of neutrinos indicate oscillations in flavor neutrinos, and hence we

can have flavor changing processes such as  $\mu \rightarrow e\gamma$ . The current upper bound on the branching ratio of this process is  $2.4 \times 10^{-12}$  at 90% C.L. [29]. For Dirac neutrinos, due to Glashow-Iliopoulos-Maiani (GIM) cancellation mechanism, the amplitude for this process is highly suppressed, and the branching ratio is well below the current upper limit. However, for Majorana neutrinos the GIM cancellation mechanism is not valid, and one may wonder if we can get appreciable branching ratio in this case. In the Majorana case, we have Type-I seesaw mechanism and in this mechanism the branching ratio for  $\mu \rightarrow e\gamma$  has been derived in [30]. For a TeV scale right-handed neutrino masses, we have found that the branching ratio of this process is  $\sim 10^{-31}$ , which is significantly smaller than the current upper limit. We believe that this small branching ratio in our model is due to small admixture between light active and heavy right-handed neutrinos.

Finally, we comment on possible restrictions on this model due to  $D^0 - \bar{D}^0$  mixing. Since the Yukawa couplings are non-diagonal in the quark sector, Higgs boson can generate flavor changing neutral current processes. Among these the Higgs couplings to up and charm quarks can give mass difference between  $D^0 - \bar{D}^0$  [7]. The current upper limit on this mass difference is  $2.35 \times 10^{-14}$  at  $2\sigma$  level [18]. In [7], the Higgs contribution to this mass difference is claimed to be  $\approx 7 \times 10^{-14}$ , which is now ruled out. However, this computation is done for a specific values of  $h_{12}^u = 1.0$  and  $h_{21}^u = 0.5$  and also for a Higgs boson mass of 200 GeV. Since now the Higgs boson has to be within 140 GeV [31, 32], we have redone the computation with  $h_{12}^u = 1.06$ ,  $h_{21}^u = 0.5$  and for a Higgs mass of 130 GeV. For  $\epsilon = 1/6.5$  and 0.16 we have found  $\Delta m_D^{\text{Higgs}} = 1.47 \times 10^{-14}$  and  $2.01 \times 10^{-14}$ , respectively. For  $\epsilon = 0.17$  we have found that the mass difference between  $D^0 - \bar{D}^0$  is  $3.26 \times 10^{-14}$  which exceeds the current upper limit. However,  $\epsilon = 0.17$  can be made allowed by choosing different set of  $h_{12}^u$  and  $h_{21}^u$  values. The price one may have to pay is adjusting the Yukawa couplings to some decimal places. Here, it can be noticed that we have fixed  $h_{12}^u$  to two decimal places to get additional suppression compared to that of [7]. The upper bound on  $\epsilon$  we are getting from  $D^0 - \bar{D}^0$  correlates with neutrino Yukawa couplings of this model. In Sec. 3 we have quoted that in the inverted hierarchical case of Majorana neutrinos,  $\epsilon = 0.17$  can give neutrino Yukawa couplings close to 1.0 rather than for  $\epsilon = 1/6.5$  or 0.16. From the current data on  $D^0 - \bar{D}^0$  mixing, the neutrino Yukawa couplings of this model are set to be on the higher side. Further improvements on the  $D^0 - \bar{D}^0$  mixing can set limits on [7] as well as neutrino sector of this model.

## 6 Conclusions

Some of the challenging problems in particle physics are the hierarchical pattern of fermion masses and the difference in the mixing pattern for quarks and leptons. A simple and elegant model [7] has been proposed to explain the hierarchies in charged



fermions and also the mixing pattern in the quark sector. One of the parameters of this model is  $\epsilon = \frac{\langle H \rangle}{M} \sim 0.15$ . After fixing this parameter and for  $\mathcal{O}(1)$  Yukawa couplings in the model, the charged fermion masses and the CKM matrix have been obtained, in agreement with experimental values.

We have generalized the model [7] to accommodate neutrino masses and also obtain mixing pattern in lepton sector, i.e. PMNS matrix. We have addressed both Dirac and Majorana masses for neutrinos in this model. To explain the Dirac and Majorana masses consistently, we have proposed dimension 20 and 12 higher order terms, respectively. These higher order terms are shown to be arriving from the UV completion of the model by having an additional flavor symmetry and some TeV scale heavy fermions and scalars. After proposing higher order terms we have done numerical analysis, where we have shown that the atmospheric and solar neutrino mass scales and mixing pattern in lepton sector can be consistently obtained for  $\epsilon$  between  $1/6.5$  and  $0.16$ . From the  $D^0 - \bar{D}^0$  mixing, we have argued that we can set an upper bound on  $\epsilon$  to be around  $0.17$ .

Since the  $\epsilon$  parameter is tightly constrained, the model has definite predictions for various collider processes. From the neutrino sector, in the Majorana case, the effective mass of the neutrinoless double beta decay,  $m_{ee}$ , has definite values in this model. The values of  $m_{ee}$ , in the inverted neutrino mass hierarchical case, are about an order less than the currently probed experimental values. This case is interesting in the ongoing and future experiments on the neutrinoless double beta decay process, to verify if the model presented here is realistic. Apart from the signals in neutrino sector, the UV completion of this model demands the presence of TeV scale weak-singlet and weak-doublet fermionic as well as singlet scalar particles in the theory. Detection of these particles is within reach of LHC experiment, which can give smoking gun signals of this model.

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